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AIRFOIL-CONTOUR MODIFICATIONS BASED ON ϵ-CURVE METHOD OF CALCULATING PRESSURE DISTRIBUTION

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ADVANCE RESTRICTED REPORT

AIRFOIL-CONTOUR MODIFICATIONS BASED ON ϵ-CURVE METHOD OF CALCULATING PRESSURE DISTRIBUTION

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SUMMARY

A method, based directly on the so-called ϵ-curve method published originally in 1931 in NACA Report No. 411, is presented for use in making modifications to the shape and pressure distribution of a given airfoil. In particular, it may be desirable to remove excessive irregularities or local peaks in the distribution. In this process it may be required that certain parameters of the airfoil be kept unchanged; for instance. the angle of zero lift, the ideal lift coefficient, or the moment coefficient. From an academic viewpoint, an altered distribution cannot be "prescribed" because compliance with the requirement of maintaining a Laplacian flow field is involved. A prescribed distribution can therefore not be obtained by iteration. The process, however adequate, is necessarily one of qualitative modifications. Several numerical examples illustrating the use of the method are given in the appendix.

INTRODUCTION

In 1931 the author introduced the so-called ϵ-curve method for calculating the pressure distribution on airfoils of arbitrary shape. (See reference 1.) The merit of this method depends essentially on the fact that the resulting integral relation can be solved by a rapidly convergent process. In the present paper the problem of effecting changes in a given pressure distribution is considered. The method is based directly on the important velocity formula (XII) of reference 1 rewritten as formula (39') in reference 2, a later report. Reference to these papers is made repeatedly herein with subsequent omission of the details concerning the use of the ϵ-curve method.

Consider a given airfoil. From the ϵ-curve of reference 1, ψ and ϵ are obtained; and from formula (XII) of reference 1 the pressure, or the equivalent velocity, is found. The velocity may be written in terms of the quantities φ, ϵ, ψ, ψ_0, $\alpha + \epsilon_T$, and v as in formula (39') of reference 2:

$$\frac{v}{V} = e^{\psi_0} \frac{\sin(\alpha + \varphi) + \sin(\alpha + \epsilon_T)}{\sqrt{\left[\sinh^2 \psi + \sin^2 (\varphi - \epsilon)\right] \left[\left(1 - \frac{d\epsilon}{d\varphi}\right)^2 + \left(\frac{d\psi}{d\varphi}\right)^2\right]}} \tag{1}$$

where the reader is referred to references 1 and 2 for the meaning of the various symbols.

Now consider a slightly altered pressure distribution. This new pressure distribution is obviously related to a new airfoil contour. It is pertinent to remark here that from a purely mathematical viewpoint the new distribution cannot be "prescribed" unless the new airfoil contour also is prescribed. In a potential flow without singularities there exists a unique relationship between the contour and the pressure distribution in the flow field. A pressure distribution cannot therefore be prescribed (mathematically) for the simple reason that the associated contour must be given in order to prescribe it. Thus the problem of specifying rigorously a pressure distribution is reduced "ad absurdum." From an academic standpoint the so-called inverse problem therefore does not exist as such.

Certain alterations of a qualitative nature may be performed in spite of the fact that a pressure change cannot be prescribed. It is the purpose of this paper to indicate a method by which qualitative alteration may be performed. It will be noted that the present method of contour modification will serve the intended purpose of the inverse method.

NATURE OF ALTERATIONS

It is useful to observe that several types of independent alteration are possible. By reference to the velocity formula (1), for instance, a change in ψ_0 will appear mainly in the multiplying factor e^{ψ_0} and will thus effect an increase or a decrease in the velocities everywhere on the contour. This change results

simply in a series of airfoils of different thickness as the major effect. It is interesting to observe that neither the angle of zero lift nor the ideal angle of attack has been changed in this operation. The quantities ϵ and ψ occurring in the velocity formula are considered available for the original airfoil contour by the ϵ-curve method of reference 1.

The effect of a change in the angle of attack α is well known and need not be discussed here. In fact, the main interest lies in improving the pressure distribution at and near the optimum, or ideal, angle of attack. In the following discussion, therefore, the proposed changes are performed at the ideal angle of attack only. In other words, the pressure distribution is examined at the ideal angle of attack, tentative changes are proposed, and results are compared at the ideal angle of attack. The restriction that the angle of zero lift remain unchanged may or may not be imposed. For airfoil contours of zero moment coefficient, as used in helicopter blades, the restriction may be imposed that the moment coefficient remain zero. In the following section the nature of such changes is examined with several types of restriction used to fulfill specified requirements. Such changes may be performed in the pressure distribution subject to any one restriction or to a combination of several simultaneous restrictions.

METHOD OF CHANGING THE ϵ-CURVE

The ϵ-curve can readily be obtained as $\epsilon(\varphi)$ by the method of reference 1. In most cases it is desirable to keep the ideal lift coefficient constant in order to obtain improvement at the exact value of the lift. Inasmuch as the expression for the ideal lift coefficient contains the factor

$$\frac{1}{2}\left(\epsilon_T - \epsilon_N\right)$$

this restriction is equivalent to maintaining a fixed difference between ϵ_N and ϵ_T, the values of ϵ at the nose and at the tail, respectively. The absolute values may or may not be kept the same. If both ϵ_N and ϵ_T are kept constant in the process of change,

the ideal lift coefficient, the ideal angle of attack, and the angle of zero lift are retained. This change is purely local and extremely restricted in nature; only minor changes will submit to this stringent type of restraint. In order to make a larger change, the condition of constant angle of zero lift may be relaxed but the requirement of a constant ideal lift coefficient retained.

An important case of alteration is the case in which the moment coefficient is kept constant. It is shown in reference 2 that the moment depends on the two lowest harmonics in the $\epsilon(\varphi)$-curve. By prescribing an alteration $\Delta\epsilon(\varphi)$ containing higher harmonics than the second the pressure distribution may be altered without affecting the moment coefficient. Here, also, further restrictions may or may not be imposed. In general, the more restrictions imposed, the more manipulations are required to adjust the ϵ-curve.

TENTATIVE PRESSURE CHANGES

How a tentative pressure change is translated into a change in the ϵ-curve will now be indicated. The $\epsilon(\theta)$- and $\epsilon(\varphi)$-curves are assumed to be available from the method of reference 1.

A pressure variation Δp along the contour may be tentatively prescribed. Since this exact change is not expected anyway, exact relationships involving Δp need not be used. It is seen from the velocity formula that $\frac{1}{2}v^2$ or its equivalent, the pressure p_s measured from the stagnation pressure, is given very nearly as

$$p_s = \frac{1}{2}\left(\frac{v}{V}\right)^2$$

$$\approx A\left(1 + 2\frac{d\epsilon}{d\varphi}\right)$$

where A is a function of position only. With similar accuracy, therefore,

$$\frac{\Delta p}{p_s} \approx 2\frac{d}{d\varphi}(\Delta\epsilon)$$

and, finally,

$$\Delta\epsilon = \frac{1}{2}\int_{\varphi_1}^{\varphi_2} \frac{\Delta p}{p_s}\, d\varphi$$

where the integral is to be taken over the range in which the tentative pressure change is given. Because this pressure change is improperly chosen, the value of

$$\Delta\epsilon = \int_0^{2\pi} \frac{\Delta p}{p_s}\, d\varphi$$

for the whole range in which the change is given will not, in general, become zero nor will the area under the $\Delta\epsilon$-curve

$$\int_0^{2\pi} \Delta\epsilon\, d\varphi$$

become zero as required by the conditions on ϵ given in reference 1. It is of paramount importance at this point to repeat that the originally prescribed pressure is necessarily unattainable, as is shown by the fact that the two foregoing integrals are, in general, different from zero. It will be noticed, however, in the following discussion that the essential "shape" effect may be retained. The process is simply to make the $\Delta\epsilon$-curve conform to the given requirements by a suitable adjustment involving the least possible change in the general form of the $\Delta\epsilon$-curve. This adjustment is made by changing the location of the maximum and minimum points on the curve or by extending the curve beyond the original range. The area under the $\Delta\epsilon$-curve can also be made zero by changing the reference or mean value. Two basic conditions must therefore be imposed on the ϵ-curve; namely, that the two foregoing integrals be zero. Several examples are treated in the appendix.

Finally, a pressure change for constant moment coefficient must be considered. It will be seen from reference 2 that the moment depends on the two lowest harmonics in the $\epsilon(\varphi)$-curve and the value of ϵ_T. The

process is as follows: Prescribe a tentative pressure change Δp, find the corresponding $\Delta \epsilon$, adjust to comply with the two basic conditions previously mentioned, and determine the following four integrals

$$A_1 = \frac{1}{\pi}\int_0^{2\pi} \Delta\epsilon \sin \varphi \, d\varphi$$

$$B_1 = \frac{1}{\pi}\int_0^{2\pi} \Delta\epsilon \cos \varphi \, d\varphi$$

$$A_2 = \frac{1}{\pi}\int_0^{2\pi} \Delta\epsilon \sin 2\varphi \, d\varphi$$

$$B_2 = \frac{1}{\pi}\int_0^{2\pi} \Delta\epsilon \cos 2\varphi \, d\varphi$$

By removing

$$A_1 \sin \varphi + B_1 \cos \varphi + A_2 \sin 2\varphi + B_2 \cos 2\varphi$$

from the initial $\Delta\epsilon$-function, the resulting $\Delta\epsilon$ is made free of the two lowest harmonics.

In general, ϵ_T will be changed slightly by the removal of the two lowest harmonics. By adding a third harmonic

$$A_3 \sin 3\varphi + B_3 \cos 3\varphi$$

both the magnitude and the slope of ϵ at the trailing edge may be left unchanged. This end is attained by choosing the proper values of A_3 and B_3. Thus, in the function

$$\Delta_1\epsilon = -A_1 \sin \varphi - B_1 \cos \varphi - A_2 \sin 2\varphi$$

$$- B_2 \cos 2\varphi + A_3 \sin 3\varphi + B_3 \cos 3\varphi$$

the constants A_3 and B_3 are determined by making

$$\Delta_1 \epsilon = 0$$

$$\frac{d}{d\varphi} \Delta_1 \epsilon = 0$$

for

$$\varphi = \pi + \beta$$

where

$$\beta = \epsilon_T$$

To the second order in β, there results for A_3 and B_3

$$A_3 = \frac{1}{3}(A_1 - 2A_2) + \frac{1}{3}(8B_1 - 5B_2)\beta + \frac{1}{3}(4A_1 - 5A_2)\beta^2 + \ldots$$

$$B_3 = B_1 - B_2 - \left(4B_1 - \frac{5}{2}B_2\right)\beta^2 + \ldots$$

Thus, the six constants are known and the desired change in the $\Delta\epsilon$-curve is given.

CONCLUDING REMARKS

It has been pointed out that the so-called inverse problem does not exist in a strict sense of the term, because a possible pressure distribution cannot be prescribed unless the new airfoil contour is actually given. An airfoil corresponding to a given pressure distribution, therefore, cannot in general be arrived at by an iteration process or by any other method. It is shown that only certain qualitative modifications may be effected. Such alterations fall logically into several independent groups. Attention is given to localized variations, in which not only the thickness factor but also the ideal angle of attack and the angle of zero lift are kept constant. Of interest, also, are the pressure changes

performed with the restriction that the moment coefficient and the angle of zero lift remain unchanged. This case is of importance for airfoils used in helicopter blades.

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APPENDIX

EXAMPLES

Five cases are treated as examples of the airfoil-contour-modification method of calculating pressure distribution:

I. R.A.F. 15 airfoil (see table I); local changes on lower surface; angle of zero lift, ideal angle of attack, and ideal lift coefficient kept constant

II. R.A.F. 15 airfoil; local changes on lower surface; moment coefficient and angle of zero lift kept constant

III. Airfoil contour generated from $\epsilon = 0.1 \sin (\alpha - 45^\circ)$, $\psi_0 = 0.1$ (see table II); ideal lift coefficient kept constant; angle of zero lift and ideal angle of attack changed

IV. Airfoil contour same as in case III; angle of zero lift kept constant

V. Airfoil contour same as in case III; restrictions same as in case I as an example of too severe restrictions

Case I is based on the R.A.F. 15 airfoil, for which figure 1 shows the shape and the pressure distribution. The purpose of the intended alteration is to remove the wavy line on the bottom surface. The first step as indicated is to draw a tentative pressure distribution. In the curve at the top of figure 2 the corresponding tentative change in pressure is plotted against the angle φ. The next step is to draw the adjusted curve for which the area

$$\int_c \frac{\Delta p}{p_s}\, d\varphi = 0$$

Thus a closed $\Delta\epsilon$-curve is assured for case I; this curve is called the adjusted curve and is shown in the center of figure 2. It is also necessary to make the area under the $\Delta\epsilon$-curve

$$\int \Delta\epsilon \; d\varphi = 0$$

This can be done readily without altering $\Delta\epsilon_N$ or $\Delta\epsilon_T$, as shown by the line for case I, for which

$$\int \Delta\epsilon \; d\varphi = 0$$

The corresponding $\Delta\psi$-curve is shown at the bottom of figure 2. The modified airfoil shape and pressure distribution for case I are shown in figure 1. Note that the pressure distribution actually obtained differs from the tentative pressure distribution and that no change has occurred in the angle of zero lift or in the ideal angle of attack.

Case II is also based on the R.A.F. 15 airfoil, with the requirement imposed that the moment coefficient and the angle of zero lift remain constant. The tentative pressure distribution is the same as that used in case I. The $\frac{\Delta p}{p_s}$-curve adjusted for zero area and the $\Delta\epsilon$-curve adjusted for zero area are therefore identical with those of case I. In this case it is necessary to comply with the requirement that the first and second harmonics be removed and that some third harmonic be added to retain the value of ϵ_T. As shown in the discussion, the function

$$\Delta_1\epsilon = -A_1 \sin\varphi - B_1 \cos\varphi - A_2 \sin 2\varphi - B_2 \cos 2\varphi + A_3 \sin 3\varphi + B_3 \cos 3\varphi$$

is to be added, where

$$A_1 = \frac{1}{\pi}\int_0^{2\pi} \Delta\epsilon \sin\varphi \; d\varphi$$

$$B_1 = \frac{1}{\pi}\int_0^{2\pi} \Delta\epsilon \cos\varphi \, d\varphi$$

$$A_2 = \frac{1}{\pi}\int_0^{2\pi} \Delta\epsilon \sin 2\varphi \, d\varphi$$

$$B_2 = \frac{1}{\pi}\int_0^{2\pi} \Delta\epsilon \cos 2\varphi \, d\varphi$$

and

$$A_3 = \frac{1}{3}\left(A_1 - 2A_2\right) + \frac{1}{3}\left(8B_1 - 5B_2\right)\beta + \frac{1}{3}\left(4A_1 - 5A_2\right)\beta^2 + \ldots$$

$$B_3 = B_1 - B_2 - \left(4B_1 - \frac{5}{2}B_2\right)\beta^2 + \ldots$$

By adding $\Delta_1\epsilon$ to $\Delta\epsilon$ of case I, the $\Delta\epsilon$-curve called case II and the corresponding $\Delta\psi$-curve in figure 2 are obtained. The resulting modified airfoil contour and pressure distribution are shown in figure 3. This case is best suited for maintaining zero moment coefficient in airfoil sections used in autogyros and helicopters.

The following three cases, cases III to V, are based on the airfoil section generated from $\epsilon = 0.1 \sin (\varphi - 45^\circ)$, $\psi_0 = 0.1$. The original and the tentative pressure distributions are shown in figure 4(a). In figure 4(b), $\Delta p/p_s$ is shown plotted against the angle φ. The tentative pressure curve is adjusted for zero area as before. The corresponding $\Delta\epsilon$-curve is marked "Case III" in figure 5 and the corresponding $\Delta\psi$-curve is marked "Case III" in figure 6. Thus far three choices have been treated:

(1) The exact shape of curve III is retained by changing the zero line, or reference line, for $\Delta\epsilon$, which changes both ϵ_T and ϵ_N but retains the difference and therefore the ideal lift coefficient. The resulting airfoil contour and pressure distribution for case III are shown in figure 7.

(2) In case IV the area under the $\Delta\epsilon$-curve corresponding to the tentative $\frac{\Delta p}{p_s}$-curve has been made zero by extending the range affected beyond the nose. In this case, only the angle of zero lift is kept constant. The result as compared with the original is shown in figure 8.

(3) In case V the restriction is purposely made too severe by specifying that no change shall occur either in the ideal lift, the angle of zero lift, or the angle of ideal lift.

Case V in figure 5 becomes distorted in attempting to fulfill the zero-area requirement and the final results shown in figure 9 are correspondingly unsatisfactory. The conclusion from this example is that, although certain requirements are desirable or required, it is not always possible to obtain a good solution within the limitations of such requirements. In such a case another basic type more suitable to the purpose must be selected.

REFERENCES

1. Theodorsen, Theodore: Theory of Wing Sections of Arbitrary Shape. NACA Rep. No. 411, 1931.

2. Theodorsen, T., and Garrick, I. E.: General Potential Theory of Arbitrary Wing Sections. NACA Rep. No. 452, 1933.

TABLE I

ORDINATES FOR R.A.F. 15 AIRFOIL AND

MODIFICATIONS, CASES I AND II

[Stations and ordinates in percent of wing chord]

Station	Upper surface			Lower surface		
	Original	Case I	Case II	Original	Case I	Case II
0	0	0	0	0	0	0
1.25	1.65	1.56	1.69	-.70	-.56	-.63
2.5	2.50	2.43	2.59	-.96	-.80	-.82
5.0	3.68	3.60	3.78	-1.19	-1.03	-.99
7.5	4.30	4.22	4.35	-1.25	-1.13	-1.00
10	4.81	4.74	4.87	-1.25	-1.23	-1.02
15	5.33	5.29	5.31	-1.05	-1.27	-.91
20	5.52	5.46	5.44	-.75	-1.25	-.81
30	5.60	5.55	5.45	-.29	-1.33	-.78
40	5.43	5.39	5.28	-.28	-1.17	-.63
50	5.09	5.05	5.00	-.47	-1.04	-.60
60	4.53	4.50	4.55	-.71	-.71	-.49
70	3.85	3.80	3.90	-.94	-.73	-.69
80	3.05	3.00	3.12	-.84	-.60	-.68
90	2.01	1.98	2.04	-.57	-.48	-.59
95	1.38	1.35	1.38	-.43	-.35	-.46
100	0	0	0	0	0	0

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[illegible]

TABLE II

ORDINATES FOR AIRFOIL CONTOUR GENERATED FROM

$\epsilon = 0.1 \sin (\varphi - 45^{\circ}), \psi_{o} = 0.1$, AND

MODIFICATIONS, CASES III, IV, AND V

Station	Upper surface				Lower surface			
	Original	Case III	Case IV	Case V	Original	Case III	Case IV	Case V
0	0	0	0	0	0	0	0	0
1.25	2.13	1.94	2.13	2.07	-1.52	-1.11	-1.07	-1.33
2.5	3.03	2.80	3.05	3.84	-2.07	-1.43	-1.39	-1.76
5.0	4.29	4.06	4.41	4.21	-2.66	-1.74	-1.63	-2.31
7.5	5.25	4.98	5.41	5.12	-2.95	-1.97	-1.76	-2.74
10	6.06	5.76	6.22	5.90	-3.10	-2.15	-1.83	-3.07
15	7.20	6.92	7.52	7.00	-3.19	-2.43	-1.97	-3.60
20	8.00	7.72	8.38	7.71	-3.02	-2.65	-2.08	-3.92
30	8.78	8.59	9.28	8.59	-2.54	-2.85	-2.17	-3.98
40	8.84	8.67	9.40	8.67	-1.86	-2.85	-2.17	-3.62
50	8.32	8.19	8.78	8.06	-1.27	-2.80	-1.94	-2.76
60	7.32	7.30	7.70	7.03	-.66	-2.41	-1.61	-2.00
70	5.87	5.90	6.18	5.67	-.24	-1.93	-1.26	-1.14
80	4.19	4.33	4.46	4.05	-.02	-1.38	-.86	-.34
90	2.32	2.41	2.48	2.13	-.04	-.82	-.54	.18
95	1.33	1.42	1.39	1.19	-.12	-.50	-.39	.16
100	0	0	0	0	0	0	0	0

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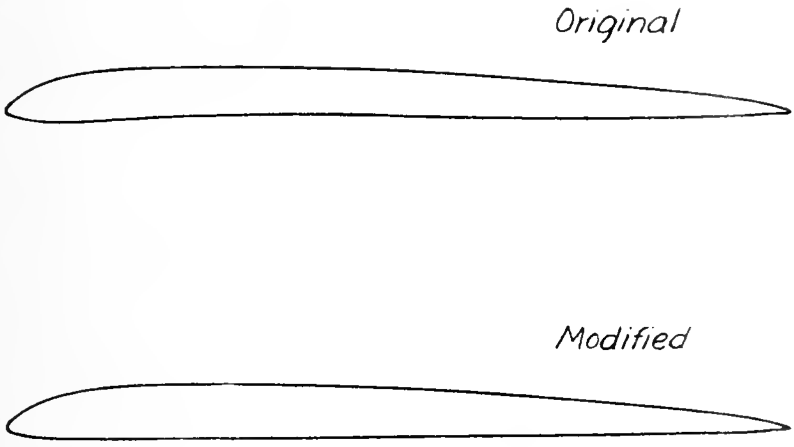

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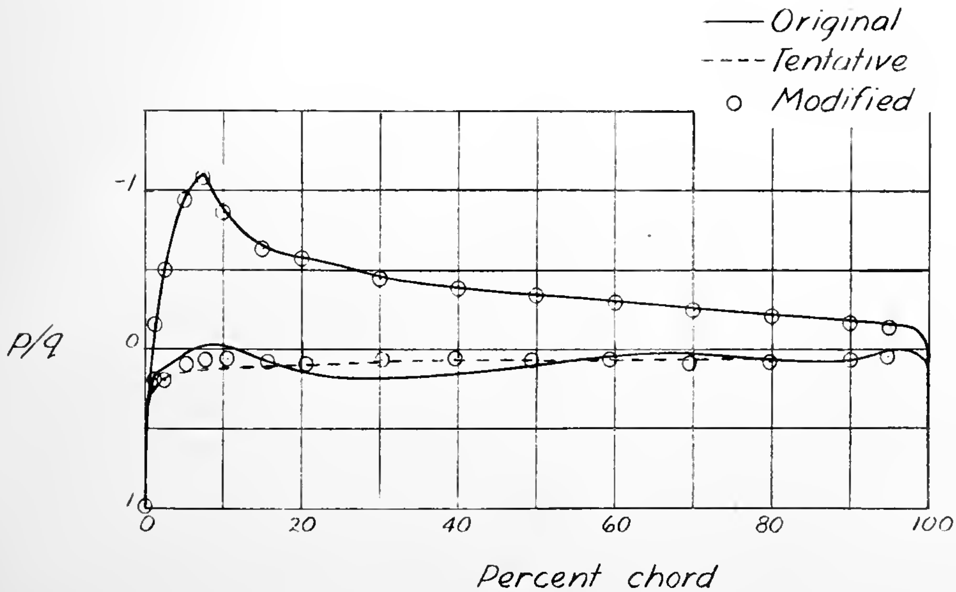

Figure 1.- Shape and pressure distribution, case I. (R.A.F. 15 airfoil)

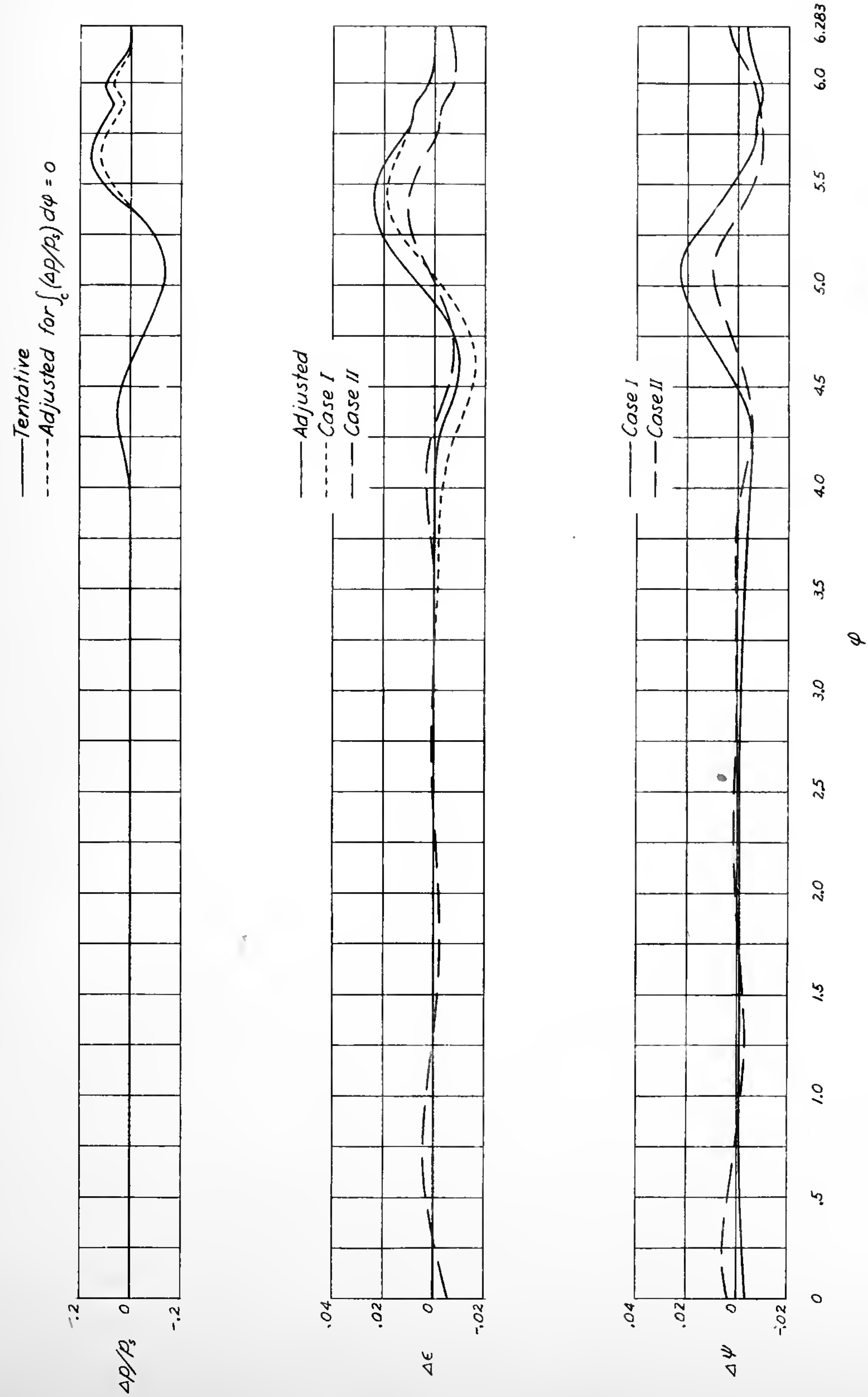

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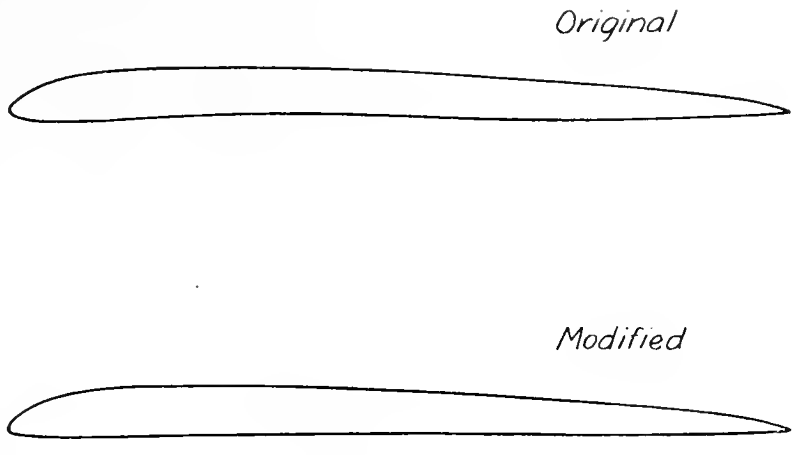
Figure 2. - Pressure, $\Delta\epsilon$-, and $\Delta\psi$-curves, cases I and II. (R.A.F. 15 airfoil.)



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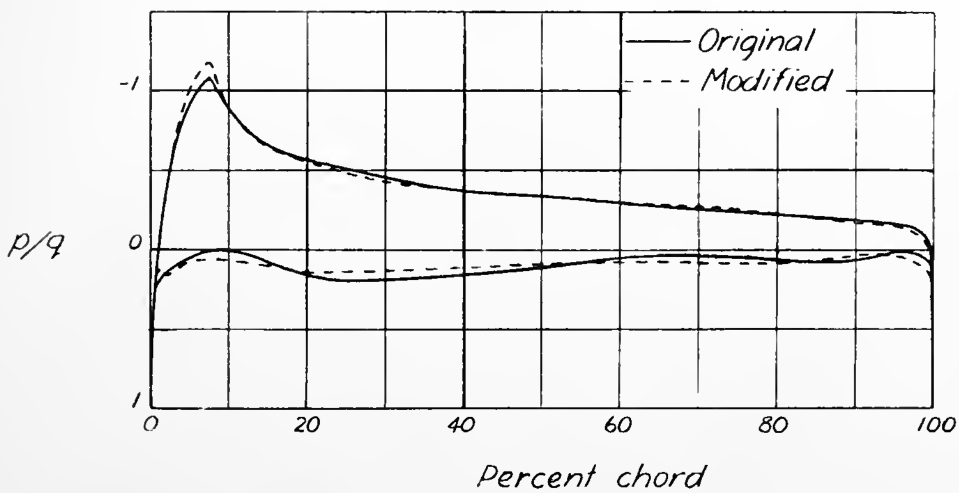
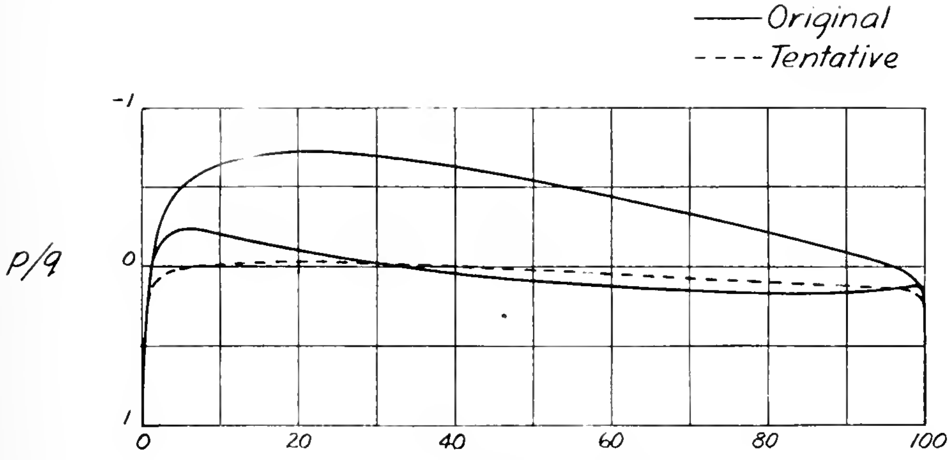
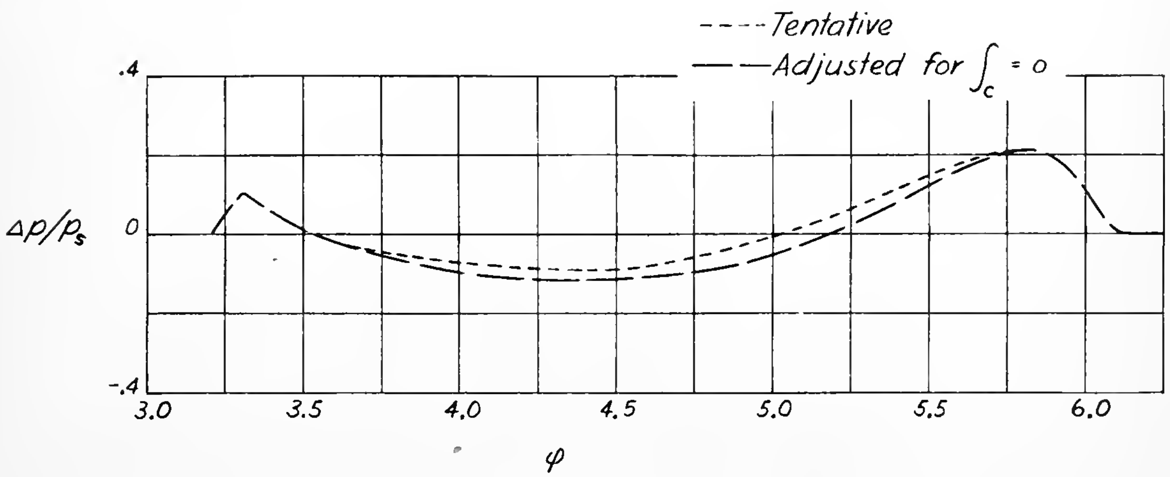


Figure 3.- Shape and pressure distribution, case II. (R.A.F. 15 airfoil.)



(a) Original and tentative pressure distributions.



(b) Tentative and adjusted pressure changes.

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Figure 4.- Pressure distributions and pressure changes, case III.

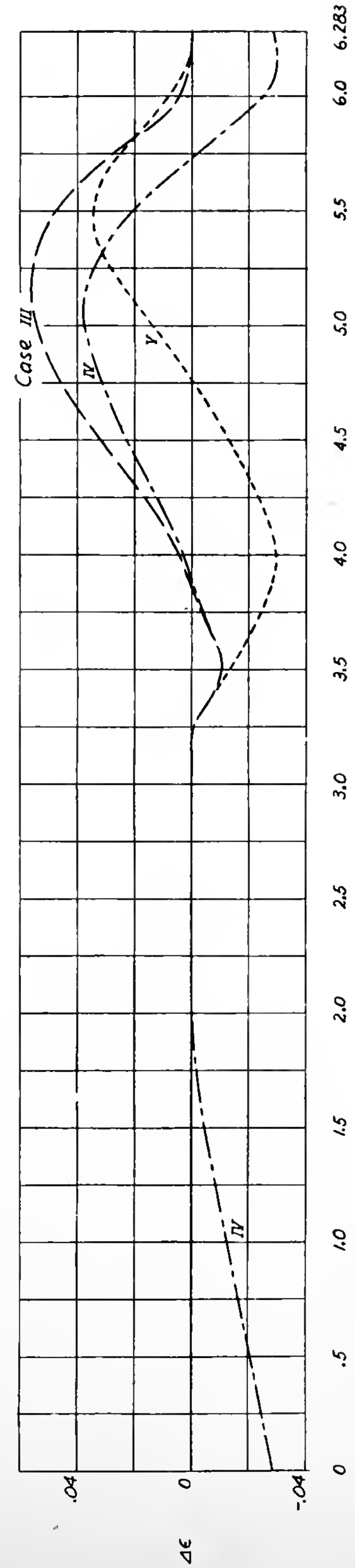

Figure 5. - $\Delta\epsilon$-curves for cases III, IV, and V.

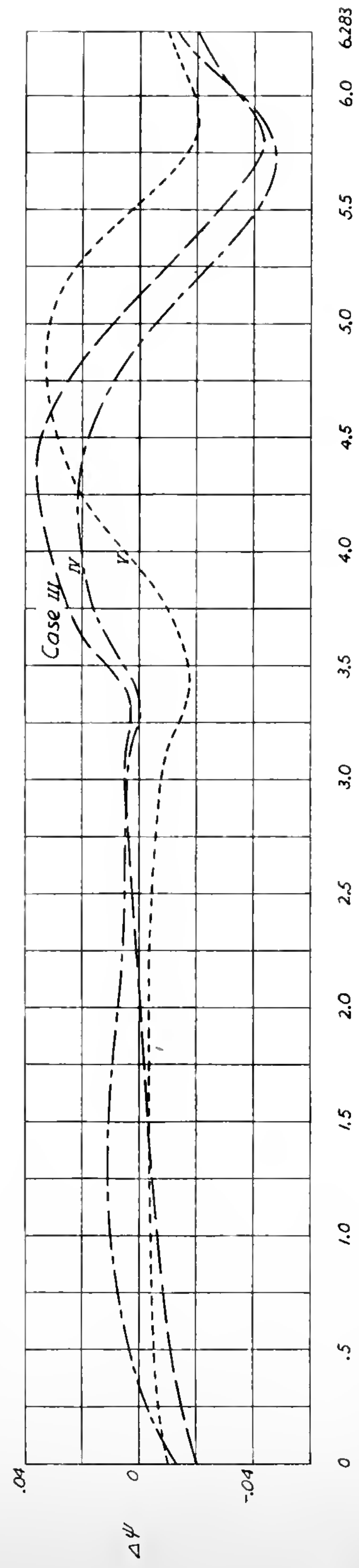

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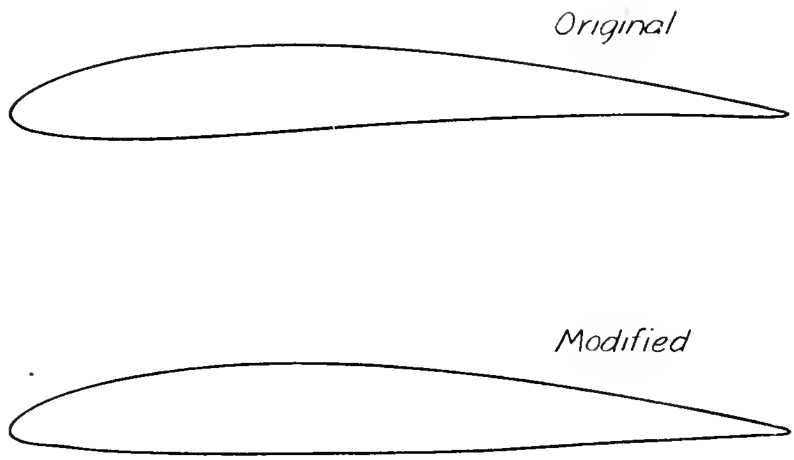
Figure 6. - $\Delta\psi$ - curves for cases III, IV, and V



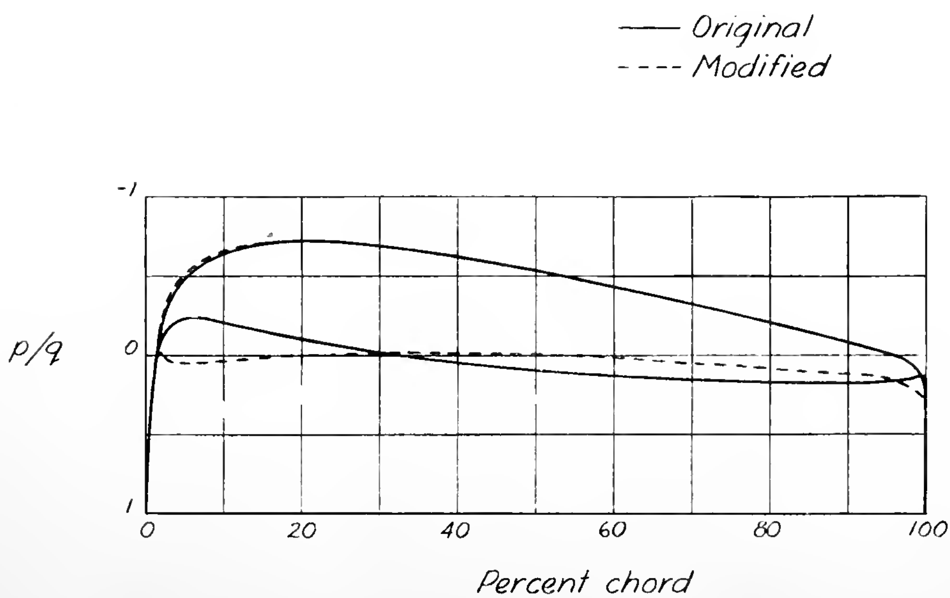
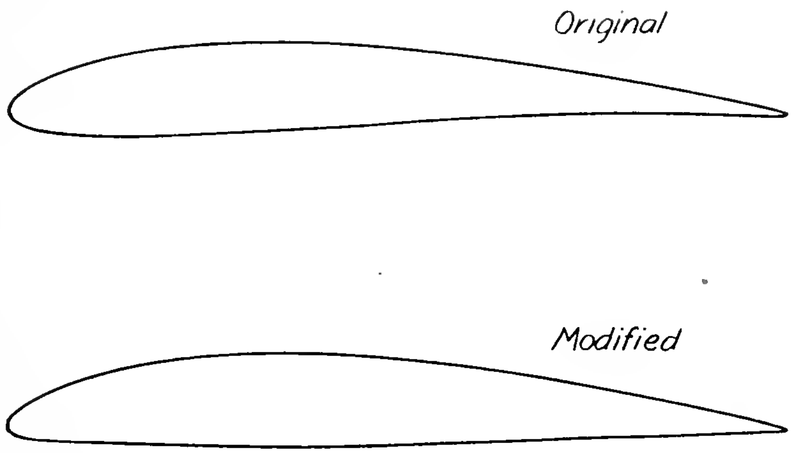


Figure 7.- Shape and pressure distribution, case III.



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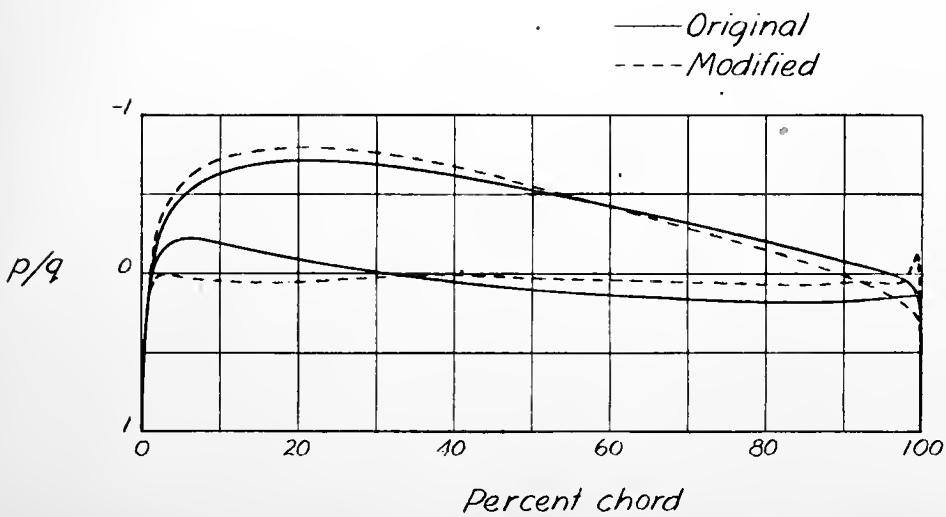

Figure 8.- Shape and pressure distribution, case IV.

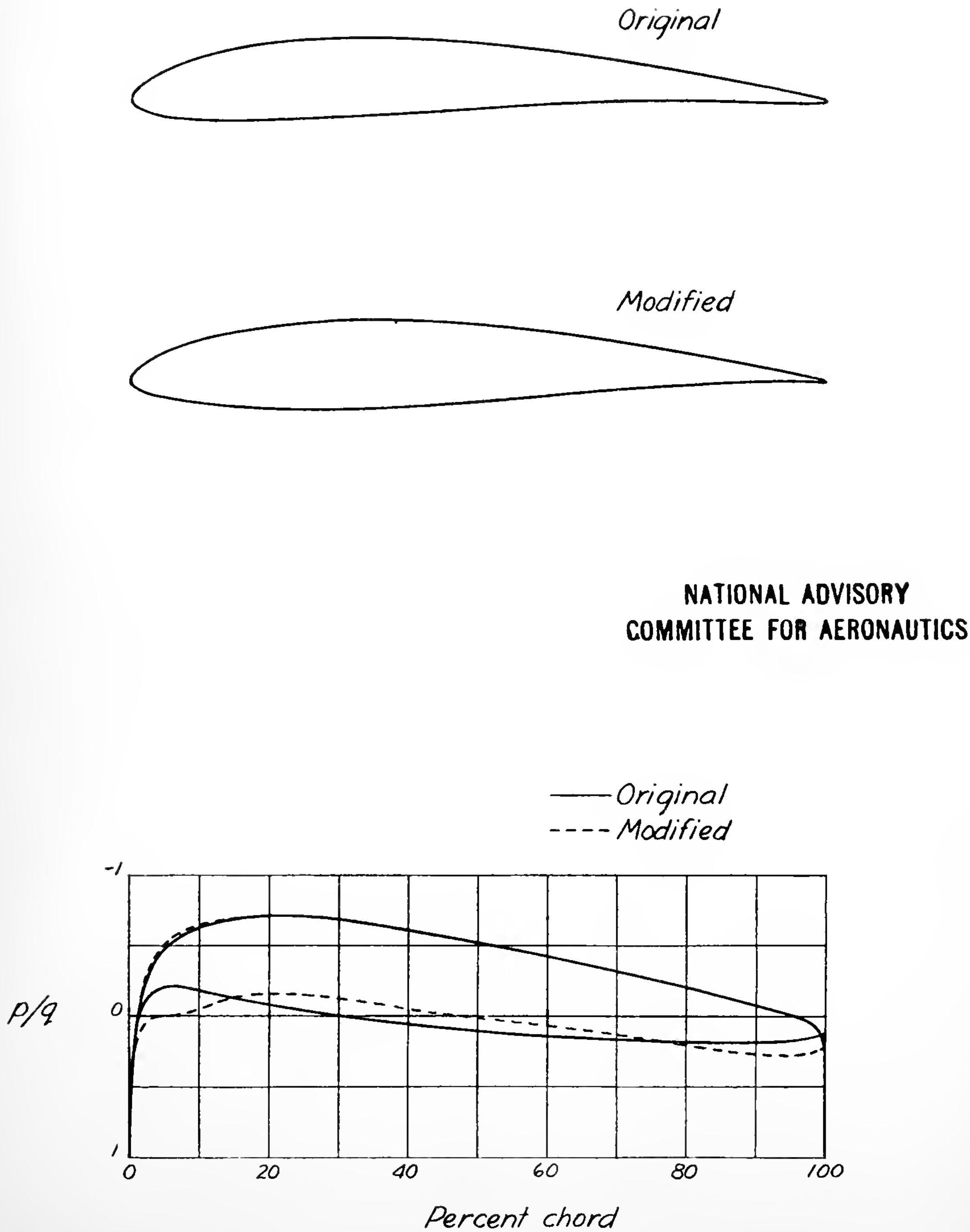

Figure 9.- Shape and pressure distribution, case V.